TELEGRAPH SPANS

IN TERMS OF

WEIGHT OF WIRE PER UNIT LENGTH.

BY

H. A. MALLOCK.

PREPARED FOR THE USE OF THE INDIAN GOVERNMENT TELEGRAPH DEPARTMENT BY ORDER OF THE DIRECTOR GENERAL.

CALCUTTA:

PRINTED AT THE GOVERNMENT TELEGRAPH PRESS, ALIPORE.

1881.

The favour of

Mr Molesworth's

criticisms

will be thankfully received

by

W H Mallock

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PREFACE

To write a complete investigation of all the mathematical theories, involved in the requirements of Spans, is a task beyond me; but, as in calculating the conditions required for a number, I have gradually collected or worked out a set of simple equations for every day use, I publish them, in the form I understand them myself, for the use of those members of the Indian Telegraph Department who may require them and may not have better information on the subject at hand.

It will be noticed that I commence, not by proving an equation for the Catenary, but by taking for granted Professor Leslie's approximate equations; one of which, *viz.* that for the strain of wire at an Insulator in a Span with points of support on the same level, was published to the Department in a Circular in 1866 and has been the basis of our work from that time.

It is to be particularly noticed that all the calculations, now given, are made, not as usual in terms of the diameters of different sizes of wire, but in terms of the weight of a unit length, and for convenience the mile is the unit chosen.

This considerably simplifies every calculation and it was with a view to this that the Specification, to which our wire is manufactured, was designed in 1872.

Some of the equations now given were originally designed by Major Eckford but have been varied so as to bring them into accord with the system of calculating by the weight of unit length.

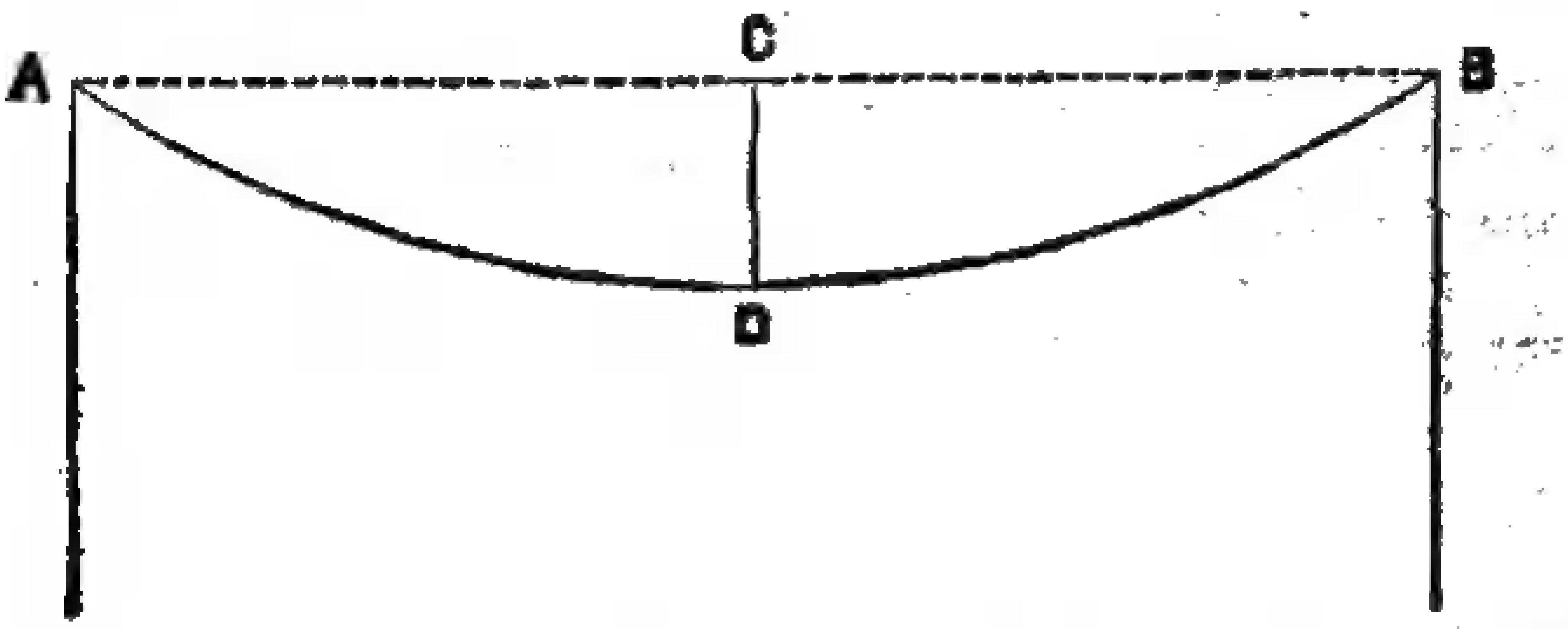
In conclusion I desire to draw attention to a far more able, but less simple paper than this, written by the late Mr. Brough and to be found in page 337 volume VI of the Journal of the Society of Telegraph Engineers, in which he worked from Gilbert's tables of the Catenary.

H. M.

SIMLA,
1st *June* 1881.

FIRST INVESTIGATION.

Spans with the points of suspension on the same level.



1. Professor Leslie's approximate equations are as follows *viz.* :—

Let

$$\left.\begin{aligned} A\,B &= a \\ C\,D &= d \\ A\,D\,B &= l \end{aligned}\right\}^{*}$$

Let then the strain at the points of suspension $= T$

" " " lowest point D $= T_2$

and the weight of a unit of length $= W$

Then

$$l = a + \frac{8\,d^2}{3\,a}$$

$$T_2 = \left\{\frac{a^2}{8\,d} + \frac{1}{6}d\right\}W$$

$$T = \left\{\frac{a^2}{8\,d} + \frac{7}{6}d\right\}W$$

or

$$\frac{T}{W} = \left\{\frac{a^2}{8\,d} + \frac{7}{6}d\right\}$$

That is to say the difference between the tensions at A and D is equal to the weight of a wire of the length $C\,D$.

2. In working out any other Equations or Calculations based on these, it must be considered

(*a*). That the *working* strain at which a wire is erected bears a certain proportion to its *breaking* strain, or that between the *working* and *breaking* strains we allow a certain margin for safety, and that in practice this safety margin is 75%, or in other words the *working strain* is ¼ *breaking strain*.

* These must always be taken in terms of the same unit of length.

(b). That the strength of wire of a similar material and quality is in proportion to its sectional area, which again varies directly as its weight per mile (or any other unit of length and weight), so that the value $\frac{T}{W}$ is constant for any size of wire of the same quality, but varies with the quality. That is, for either, Unannealed Iron wire, Bessemer Steel wire, or Cast Steel wire, it would be higher because the breaking strain is higher.

Hence $\frac{T}{W}$ may be termed the "Constant of the Wire," whilst $\frac{1}{4}$ of $\frac{T}{W}$ is the Constant Working Strain at the Insulator.

3. For the Iron wire we use the Contract Breaking Strains, with the minimum ductility allowed, are

For wire weighing per mile lbs.	Contract Breaking Strain lbs.
900	3075
750	2562
600	2050
450	1537
300	1025

4. From which it will be seen that if, for the wire we now have we calculate out a Span in yards, the "Constant of Wire" or $\frac{T}{W} = \frac{2050}{600} \times 1760 = 6013$ yards = say 6000 yards or 18000 feet.

Whilst the Strain at the points of suspension, that is at the Insulators is $\frac{1}{4}$ of 6000 yards = 1500 yards or 4500 feet, and as the Cast Steel Strand we use has 3 times the strength of Iron wire its constant breaking strain = $\frac{1}{4}$ of 18000 yards. = 4500 yards.

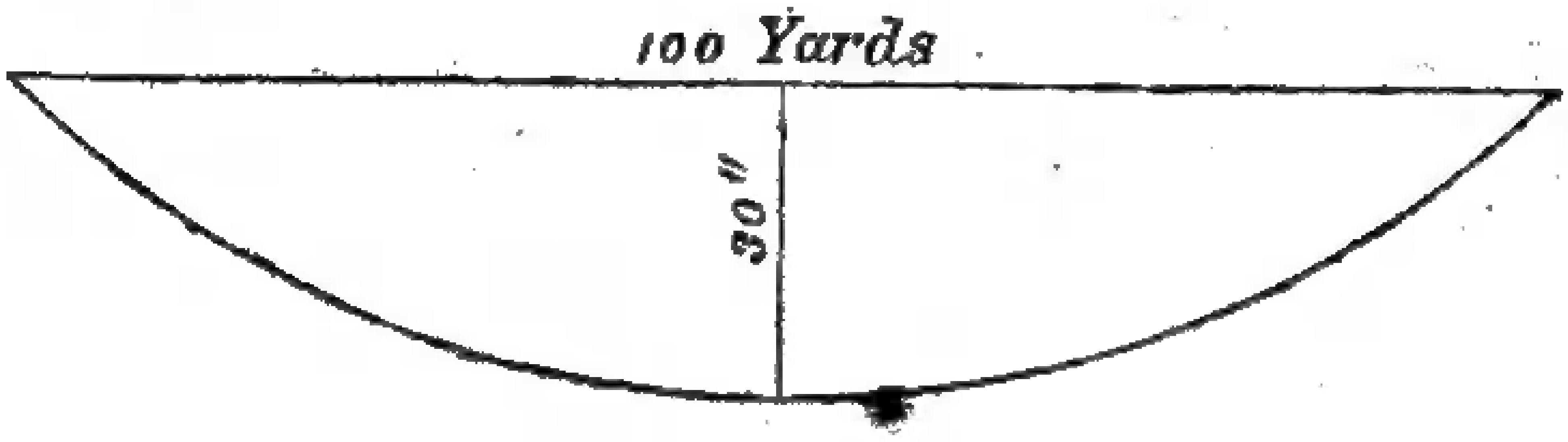
5. It is well known that *for ordinary Line wire a dip* in the proportion of 30 inches to a span of 100 yards is successful, and the following simple equation will show how that dip agrees with the above formula of

$$\frac{T}{W} = \frac{a^2}{8d} + \frac{7}{6}d$$

a = span and *d* = dip.

In spans of under 400 yards it will be seen on a trial calculation that the item $\frac{7}{6}d$ may be disregarded, as it is really (d being the weight of a length of wire equal to the dip) only $\frac{7}{6}$ of the weight of 30 inches of wire, in a Span of 100 yards.

Hence allowing $\frac{1}{4}$ breaking strain for the working strain we get



$$\frac{1}{4}\,\frac{T}{W} = \frac{100^2}{8 \times \frac{30}{36}}$$

(Since $\frac{30}{36}$ of a yard = 30 inches)

$$\text{Then } \tfrac{1}{4} \text{ of } 6000 = \frac{12000}{8}$$

or $$1500 = 1500$$

Q. E. D.

6. To find the dip the wire should have.

$$T = \left(\frac{a^2}{8\,d} + \frac{7}{6}\,d\right) W.$$

$$\frac{T}{W} = \frac{3\,a^2 + 28\,d^2}{24\,d}$$

$$\frac{T}{W} \times 24\,d = 3\,a^2 + 28\,d^2$$

$$28\,d^2 - 24\,\frac{T}{W}\,d = -\,3\,a^2$$

$$d^2 - \frac{24\,\frac{T}{W}\,d}{28} = -\,\tfrac{3}{28}\,a^2$$

$$d^2 - \tfrac{6}{7}\frac{T}{W}d + \left(\tfrac{3}{7}\frac{T}{W}\right)^2 = \left(\tfrac{3}{7}\frac{T}{W}\right)^2 - \tfrac{3}{28}a^2$$

$$d - \tfrac{3}{7}\frac{T}{W} = \sqrt{\left(\tfrac{3}{7}\frac{T}{W}\right)^2 - \tfrac{3}{28}a^2}$$

$$= \pm\sqrt{\tfrac{9}{49}\frac{T^2}{W^2} - \tfrac{3}{28}a^2}$$

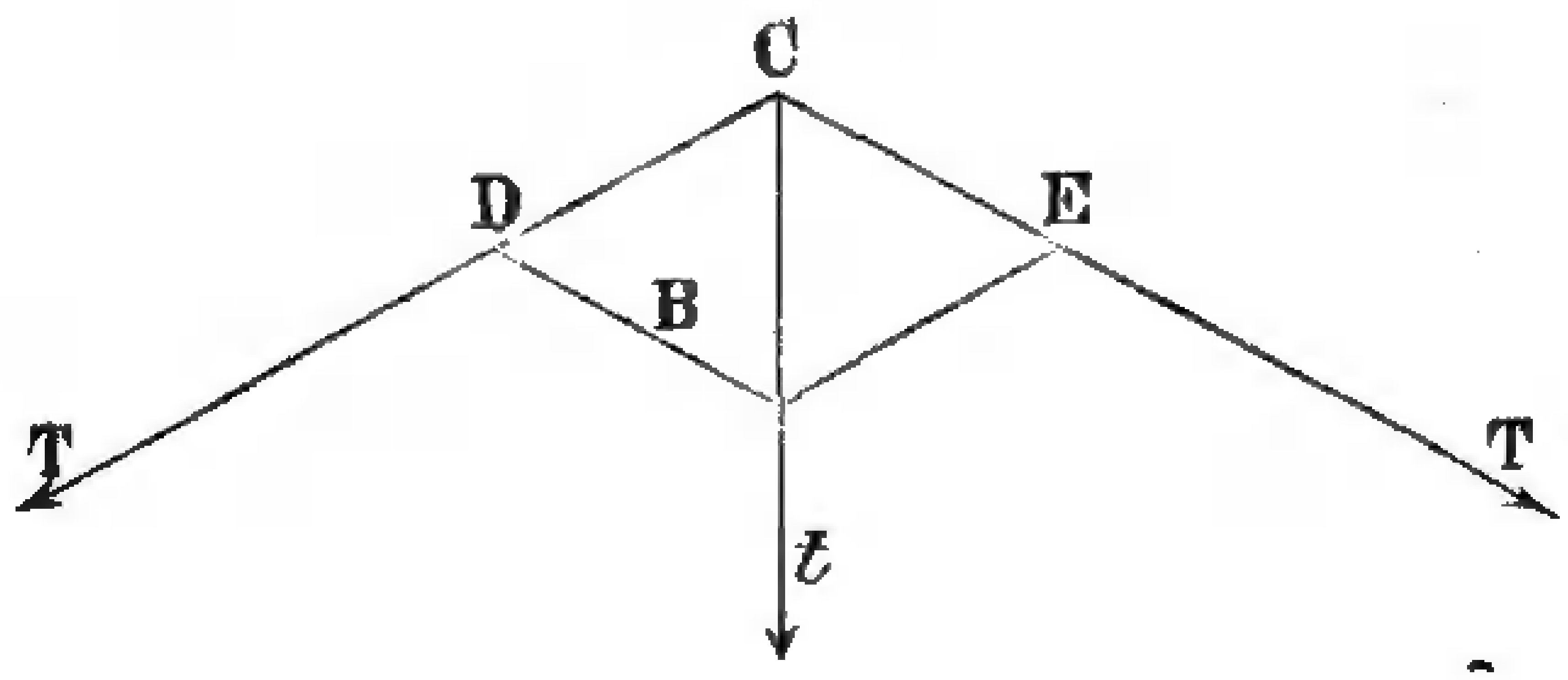
$$= -\sqrt{\frac{36\frac{T^2}{W^2} - 21a^2}{196}}$$

$$= -\frac{\sqrt{36\frac{T^2}{W^2} - 21a^2}}{14}$$

$$d = \tfrac{1}{7}\left(\frac{3T}{W} - \frac{\sqrt{36\frac{T^2}{W^2} - 21a^2}}{2}\right)$$

Note.—In calculating this it must be remembered to take $\frac{T}{W}$ at the *working* strain of the wire.

7. To find the strain on an Insulator at an angle.



Let, $C\ D$ and $C\ E$ be two Telegraph wires.
T, T the strains on them.
t resultant strain at C.
$\theta =$ angle $D\ C\ E$.
$B =$ angle $t\ C\ D = \frac{D\ C\ E}{2} = \frac{\theta}{2}$

Here as,

$$T : t :: \sin. B : \sin. D.$$

$$:: \sin. B : \sin. (180-2 B.)$$

$$:: \sin. B : \sin. 2 B.$$

$$:: \sin. B : \sin. B \cos. B.$$

$$1 : 2 \cos. B.$$

$$t = 2 T \cos. B.$$

$$= 2 T \cos. \frac{\theta}{2}$$

8. To find the angle included by two wires, the strength of the insulator being fixed.

From No. 7 $\cos. \frac{\theta}{2} = \frac{t}{2 T}$

$$= \frac{t}{2 W\left\{\frac{a^2}{8 d} + \frac{7}{8} d\right\}}$$

9. But in practice $\frac{7}{8} d$ will be immaterial in respect to the other terms of the Equation, and may be disregarded, when the simplified Equation will stand

$$\cos. \frac{\theta}{2} = \frac{t}{2 W\left(\frac{a^2}{8 d}\right)}$$

$$= \frac{4 t d}{W a^2}$$

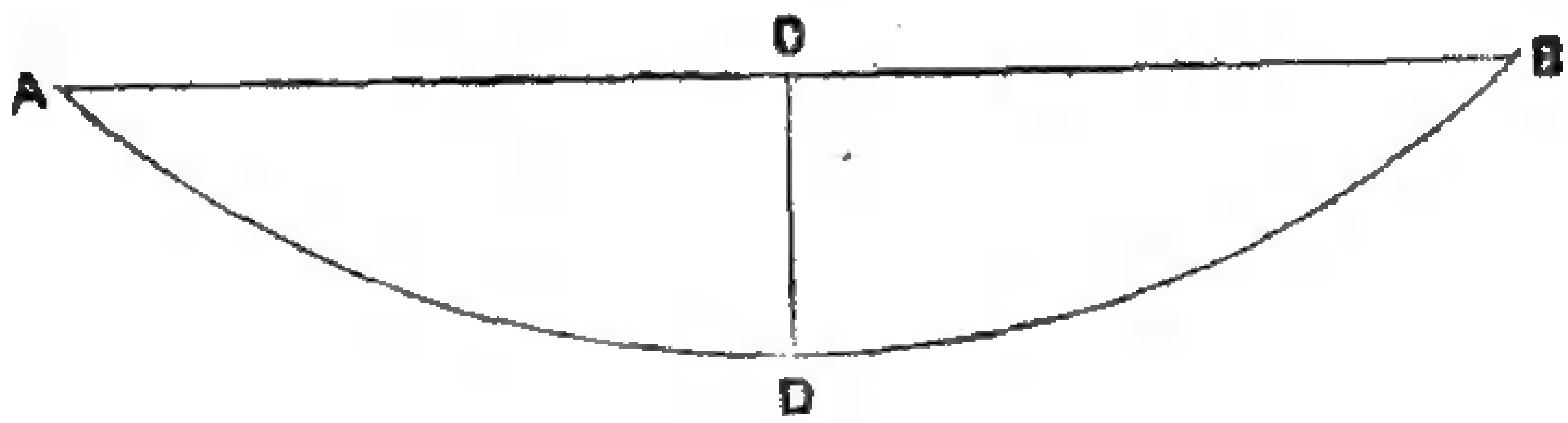
SECOND INVESTIGATION.

Spans with the points of suspension on different levels.

1. To work out these from the Catenary would be excessively difficult, but (by, as above, ignoring the $\frac{7}{6}d$) we may from the Parabola, find the position of a vertical line passing through the lowest point in the Span, and having got that afterwards get the proper dip from the Catenary equation.

2. That the calculations may be clear I must first of all show how (if we omit the $+\frac{7}{6}d$) the ordinary equation for the Parabola will agree with the equation of

$$\frac{T}{W}=\frac{a^2}{8d}$$



3. In the ordinary Parabola equation

let $\qquad p =$ parameter

then $\qquad A\,C^2 = p\,(C\,D)$

but $\qquad A\,C = \frac{1}{2}\,A\,B$

$\therefore \qquad \dfrac{A\,B^2}{4} = p\,(C\,D)$

or $\qquad A\,B^2 = 4\,p\,(C\,D)$

4. But (omiting the $\frac{7}{6}d$)

$$\frac{T}{W}=\frac{A\,B^2}{8\,C\,D}$$

or $\qquad A\,B^2 = \dfrac{T}{W}(8\,C\,D)$

then substituting from above

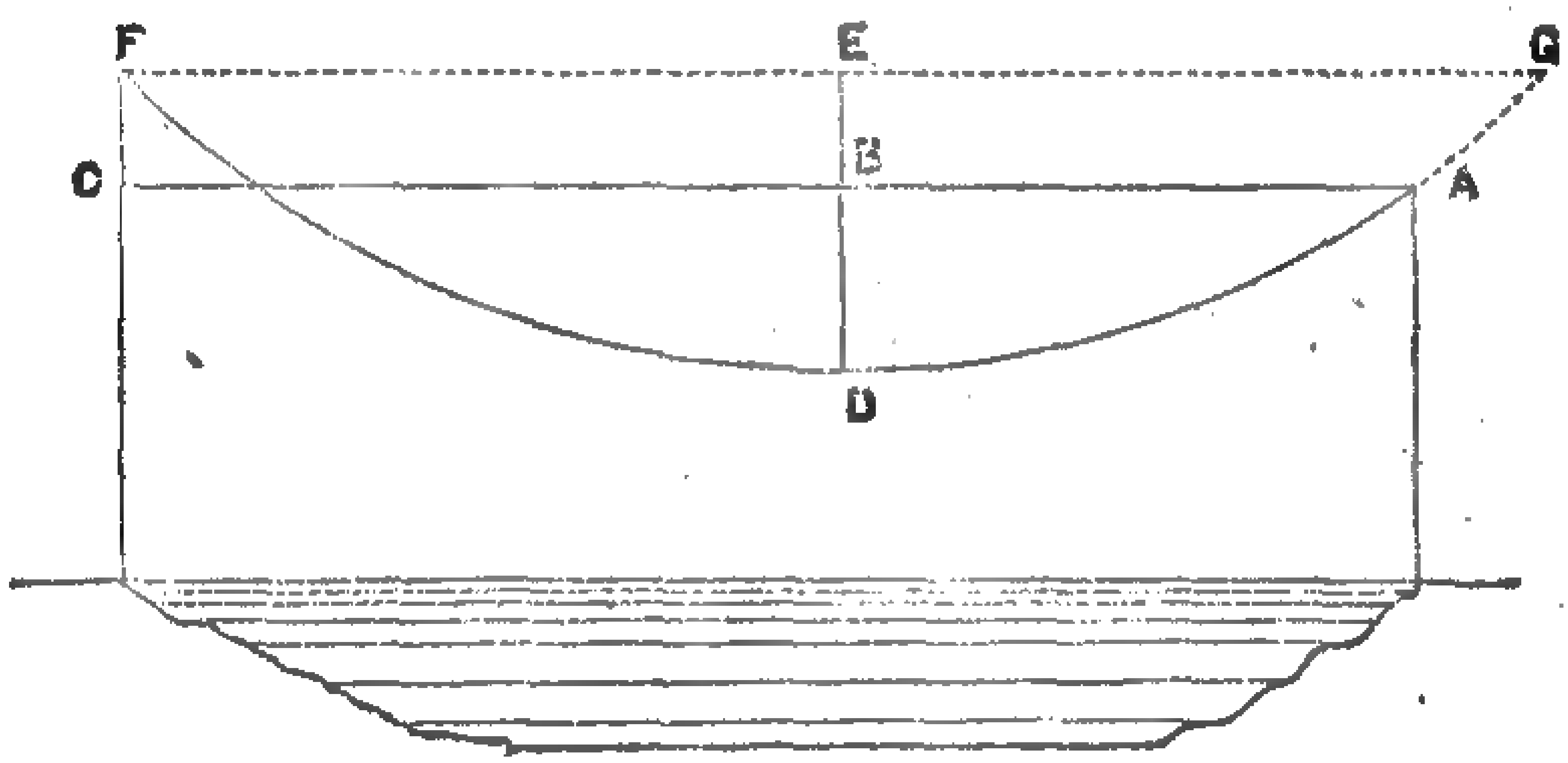
$$\frac{T}{W}(8\,C\,D) = 4\,p\,(C\,D)$$

$$8\,\frac{T}{W} = 4\,p$$

$$p = 2\,\frac{T}{W}$$

or the Parameter = 2 the Working Strain of the wire. That is to say, if the wire be erected with 75% margin of *safety*, and be supposed to lie in the form of a Parabola, the Parameter of that Parabola = 2 × 1500 = 3000 yards.

5. With this preface we now come to the equation for Spans with points of support at different levels.



Let
$$A\,B = x$$
$$B\,C = y$$
$$B\,D = d$$
$$B\,E = a$$
$$A\,C = l = x + y$$
$$p = \text{parameter}$$
$$= \text{(see above) 2 Constant.}$$

Then in the Parabola $A\,D\,F$

$$A\,B^2 = p\,(B\,D)$$
$$= p\,d$$

or

$$x^2 = p\,d \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$B\,C^2 = p\,(D\,E)$$

or
$$y^2 = p\,(a+d) \qquad \ldots \qquad \ldots \quad (2)$$

$$\frac{y^2}{x^2} = \frac{p\,(a+d)}{p\,d} = \frac{a+d}{d} = 1 + \frac{a}{d} \ldots \qquad \ldots \quad (3)$$

but
$$y = l - x$$

Hence, from (3)

$$\frac{(l-x)^2}{x^2} = 1 + \frac{a}{d}$$

$$l^2 - 2\,l\,x + x^2 = x^2 + \frac{a\,x^2}{d}$$

$$l^2 - 2\,l\,x = \frac{a\,x^2}{d}$$

and substituting, from (1), $p\,d$ for x^2

$$l^2 - 2\,l\,x = \frac{a\,p\,d}{d}$$

$$= a\,p$$

$$2\,l\,x = l^2 - a\,p$$

$$x = \frac{l^2 - a\,p}{2\,l} \qquad \ldots \qquad \ldots \quad \mathrm{I}$$

Again
$$\frac{x^2}{y^2} = \frac{p\,d}{p\,(a+d)}$$

but
$$x = l - y$$

$$\therefore \qquad \frac{l^2 - 2\,l\,y + y^2}{y^2} = \frac{d}{a + d}$$

Substituting $p\,(a+d)$ for y^2

$$l^2 - 2\,l\,y + p\,(a+d) = p\,(a+d)\left(\frac{d}{a+d}\right) = p\,d$$

$$l^2 - 2\,l\,y = p\,d - p\,a - p\,d$$

$$2\,l\,y - l^2 = p\,a$$

$$2\,l\,y = l^2 + p\,a$$

$$y = \frac{l^2 + p\,a}{2\,l} \qquad \ldots \qquad \ldots \quad \mathrm{II}$$

6. These two equations give us the lengths $A\,B$ and $B\,C$ or the point B.

7. We must then double $B\,C$ or $E\,F$ to get the point G the end of an imaginary Span with both points of suspension at the same level.

8. Then from the Catenary Equation for the dip (see No. [illegible] of first section.

$$D\,E = \frac{1}{7}\left(\frac{3\,T}{W} - \frac{\sqrt{36\,\frac{T^2}{W^2} - 21\,F\,G^2}}{[illegible]}\right)$$

9. But $D\,E = d + a$

and as a is known we get d.

THIRD INVESTIGATION.

Back Spans, or Balancing Spans.

1. After the considerations for the Main Span come those for the Spans immediately behind it, which must be so designed that (*I*) the wire must rest naturally on the Mast without any strain on the Insulator in either direction, (except that which is due to the difference of wind pressure on the different lengths of wire) and on the post next the Mast with no greater strain than the Insulator can well bear, (*II*) the wire must lie on the latter by its own weight and not be dragged down to it, and (*III*) by gradually altering the size of the wire used, the strain, if too much for any one Insulator may be divided amongst several.

2. For the first and second we must consider how the wire is to run free over the Masts of the Span, and then to fall naturally on to what is commonly called the "Terminal Post." *This name of "Terminal Post" being very familiar it has been adhered to, but it must be understood that this post does not terminate all the arrangements for balancing the strain on the Span, for these arrangements may have to run back for some distance.*

3. As there must be no strain on the Insulator on the Span Mast, the tension of the Span wire due to its weight and the dip at which it is erected, must be balanced by an exactly equal tension behind the Mast.

4. If the wire in the Span between the Mast and the "Terminal Post" or the "First Back Span," were the same size as the Span wire and at an equal tension, it would necessarily have the same dip, which could only be arranged by making the (so called) "Terminal Post" very high. Indeed, supposing the distance of the "Terminal Post" from the Mast were $= \frac{1}{2}$ the span, then the height of the "Terminal Post" would be the height of the Mast less the dip of the Span, and altogether we should get impracticable requirements to carry out.

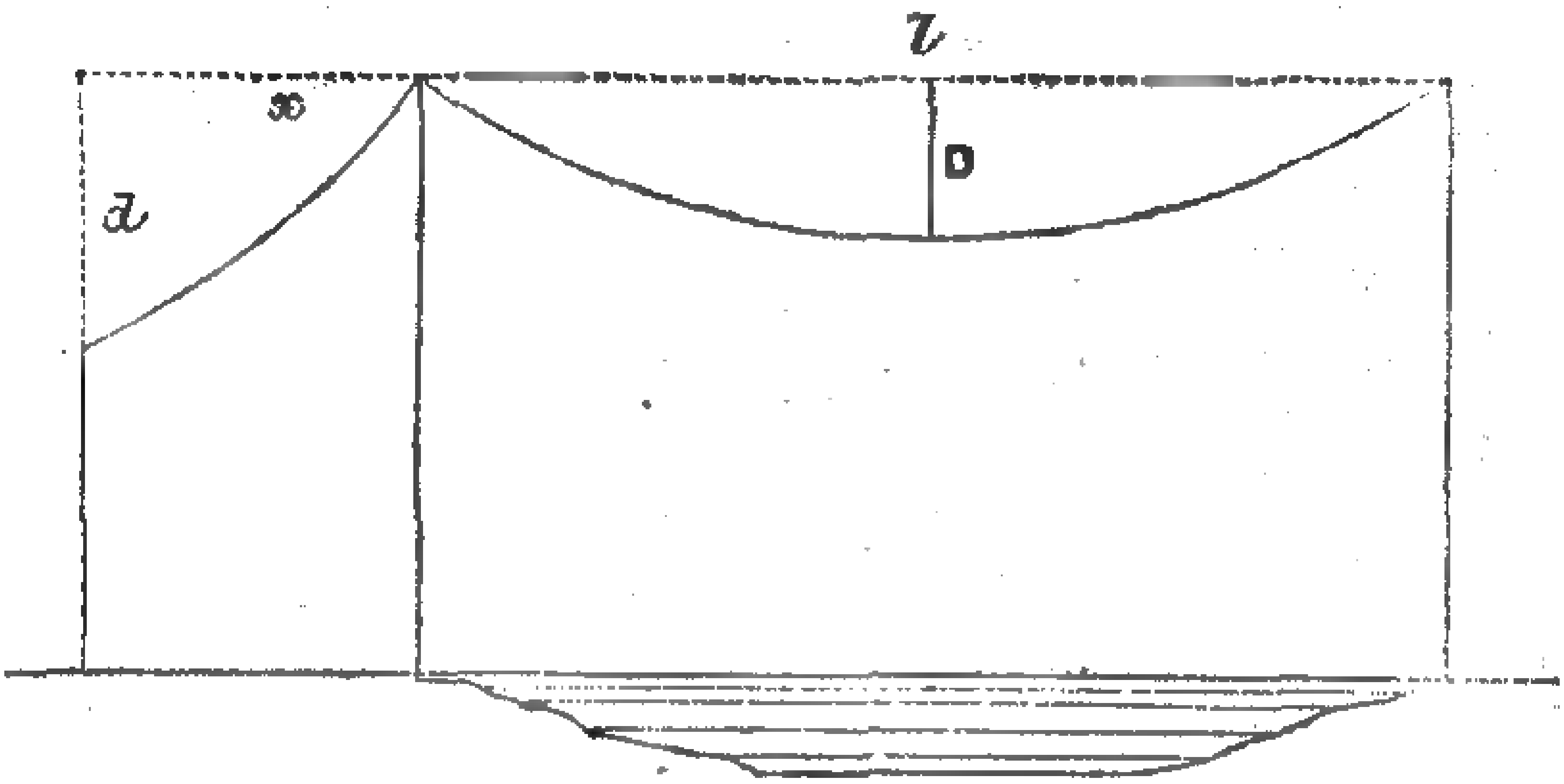
5. The dip of the wire however varies, *cæteris paribus*, directly as its weight, so by varying the length and weight of wire in the "First Back Span" we can by it so balance the "Main Span" wire that there shall be no strain on the Span Mast Insulator.

6. The following equation gives all the factors of the "First Back Span" and from it we can vary either

(1). The distance of the "Terminal Post" from the Mast.

(2). The height of the "Terminal Post."

(3). The size of wire.



Let l = length of Main Span in yards.

D = dip of Main Span in feet.

x = length of Back Span in yards.

d = dip „ „ feet.

= also the difference in height between the Main Span Mast and "Terminal Post."

n = the proportion of weight per mile in Back Span wire to weight per mile of Main Span wire, that is if W = weight per mile in Back Span and w the weight per mile in Main Span then $n = \frac{W}{w}$

a = the proportion of dip in feet at which the main Span wire is calculated, thus if the dip of the main Span wire be 10″ per 100 yards, then $a = \frac{10}{12}$, or if it be erected at 1 foot per 100 yards

then $a = \frac{12}{12} = 1$

Now $\quad l^2 : 100^2 :: D : a$

$$l^2 = \frac{(100^2)\, D}{a}$$

But $\quad l^2 : (2\,x)^2 :: D : \frac{d}{n}$

$$(2\,x)^2 = l^2 \times \frac{d}{n\,D}$$

(Substituting value of l^2 given above)

$$(2\,x)^2 = \frac{(100^2)\,D}{a} \times \frac{d}{n\,D}$$

$$= \frac{(100^2)\,d}{a\,n}$$

$$2\,x = 100\sqrt{\frac{d}{a\,n}} \text{ and } x = 50\sqrt{\frac{d}{a\,n}}$$

7. As all our wire is made in multiples of a unit weight per mile, every size bears a simple proportion to any other size; and, [illegible] it is easy to twist up either a 3 Strand or a 7 Strand rope of any size wire, the variation of weight, when we know the proportion we want, becomes a very easy matter.

8. In practice however for all Spans that require Steel wire we generally use 450 lbs. Steel Strand, and the most convenient Balancing Span wire is a 3 Strand 450 lbs. per mile Iron wire, because being 3 times the weight of the Steel wire it will take 3 times the dip, and as Iron wire has [illegible] "Constant Strength" equal to [illegible] weight of 6000 yards of itself, whilst the "Constant Strength" of Steel wire equals 18000 yards of itself, when the 3 Strand 450 lbs. Iron wire balances up the 450 lbs. Steel wire to its proper dip, both will be working at the same proportion of their strength.

FOURTH INVESTIGATION.

Second Back Spans.

1. The next point to be considered are the proportions the Second Back Span should bear to the "Balancing Span" so that the strain on the Insulator of the (so-called "Terminal Post") may be reduced by another somewhat lesser Strain behind, and that the first Strain may be no greater than the Insulator can well bear.

2. Assuming the posts at each end of the Second Back Span to be equal in height we want to reduce the Strain on the "Balancing Span."

3. Supposing the Balancing Span to have in it a 3 Strand rope of 450 lbs. Iron wire, the Strain on the Insulator at a working Constant of 1500 yards would be

$$\frac{1500}{1760} \times 3 \times 450 \text{ lbs.}$$

Then if in the Second Back Span we use 900 lbs. wire, the Strain on the other side of the Insulator would be

$$\frac{1500}{1760} \times 900 \text{ lbs.}$$

So that the Resultant Strain on the Insulator becomes

$$\left\{(3 \times 450) - 900\right\}\frac{1500}{1760}$$

$$= 450 \times \frac{1500}{1760}$$

$$= 383\tfrac{92}{176}$$

or say 384 lbs.

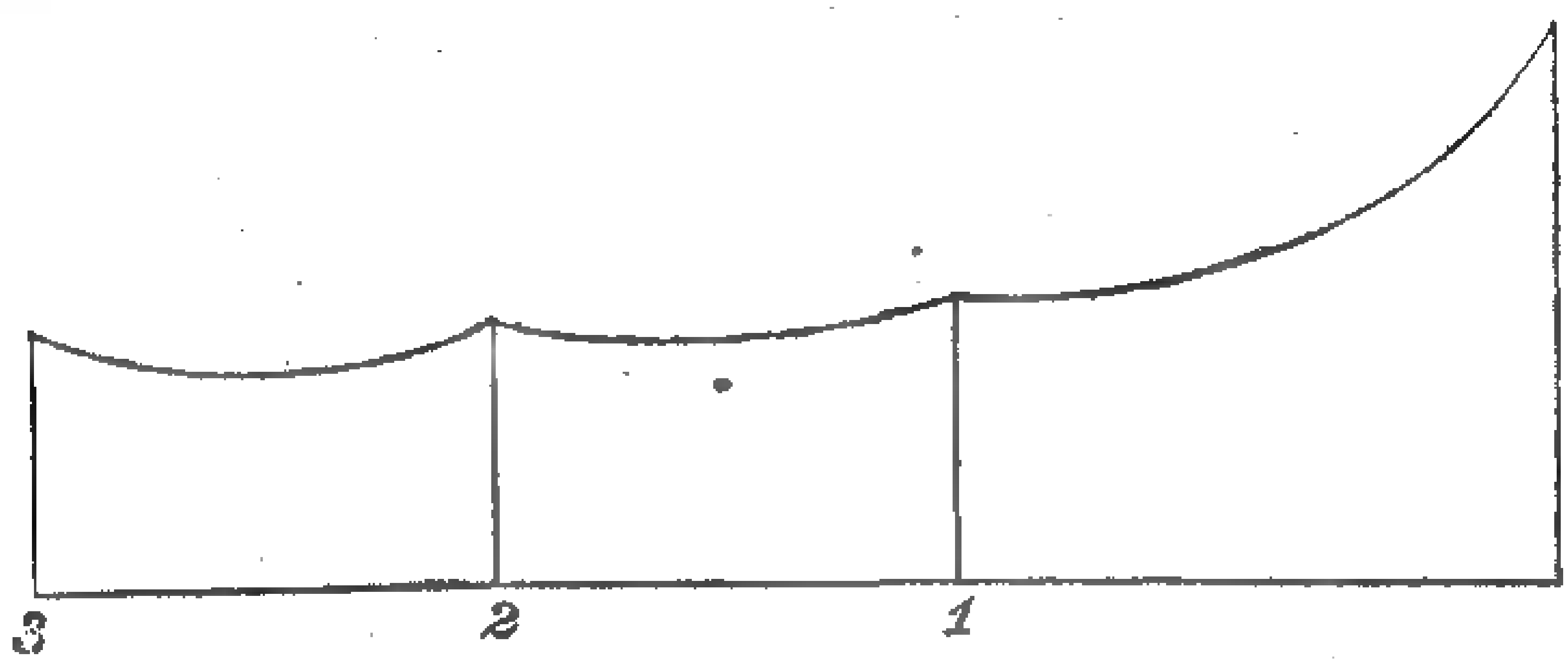
4. We can also reduce the Strain on the (nominal) Terminal Insulator by varying the dips of the wires somewhat as follows *viz.*

Supposing we give the Main Span Steel wire a dip in the proportion of 1 foot per 100 yards, and the "First Back Span" (made of Iron wire 3 times the weight of that in the Main Span) a dip at the rate of 3 feet per 100 yards, then the Strain on the Span side of the Terminal Insulator would be

not $$\frac{1500}{1760} \times 3 \times 450$$

but $$\frac{30}{\ } \times \frac{1500}{\ } \times 3 \times 450$$

5. If the Line wire be generally 600 lbs. per mile, then after erecting one Span of 900 lbs. wire, if the 600 lbs. wire comes into the next Span the resultant Strain to the left at post No. 2 below—



would be

$$(900 - 600)\frac{1500}{1760}$$

6. It is to be here noted that, for all ordinary purposes of calculation, if wire which has a "Constant Strength" of 6000 yards of its own length, be erected at $\frac{1}{4}$ breaking strain or 1500 yards, then its working strain at the Insulator which is $\frac{1500}{1760}$ of its weight per mile may be sufficiently accurately calculated at $\frac{1500}{1800}$ or $\frac{5}{6}$ of its weight per mile, which is a very simple proportion to remember.

FIFTH INVESTIGATION.

To find the Wind pressure on a wire in terms of its length.

(1). The wire being a Cylinder the pressure is assumed to be $\frac{2}{3}$ of what it would be on a flat surface.

(2). A rod of iron 1 inch in diameter and 1 mile long weighs 13833·6 lbs.

(3). Hence the diameter of any size wire in inches is

$$\sqrt{\frac{\text{weight per mile in lbs.}}{13833{\cdot}6}}$$

(4). Assuming the Wind Pressure per sq. foot to be p pounds and the total pressure on any length l feet to be P pounds, the pressure on 1 foot of length and d inch diameter will be

$$p \times \frac{2}{3} \times \frac{1}{12} \times d$$

and for any length of l feet

$$P = p\,l \times \frac{1}{18} d$$

substituting from (3)

$$P = p\,l \times \frac{1}{18} \sqrt{\frac{\text{weight per mile in lbs.}}{13833{\cdot}6}}$$

$$= p\,l \times \frac{1}{18} \times \frac{\sqrt{w}}{117{\cdot}62}$$

$$= \frac{p\,l}{2117{\cdot}16} \sqrt{w} \text{ lbs.}$$

(5). In order to get the *exact* number of feet of wire in a Span we might work from the first of Professor Leslie's equations given on page 1 which is as follows :—

where $a =$ Span in yards

$d =$ dip in yards

$l =$ length of curve in yards

then $$l = a + \frac{8\,d^2}{3\,a}$$

6. But since on working this out for a Steel wire Span of 1000 yards we find that

$$l = 1002{\cdot}136$$

it is evident that for the flat curves we use for Steel wire the extra length due to the curve may be disregarded, and to facilitate the calculation we may assume the wind pressure

SIXTH INVESTIGATION.

Additional Strain on the wire due to Wind Pressure.

In Spans with points of Support on the same level the additional strain on the wire at each side of the Insulator due to Wind Pressure may be calculated on a length of wire equal to half the Span.

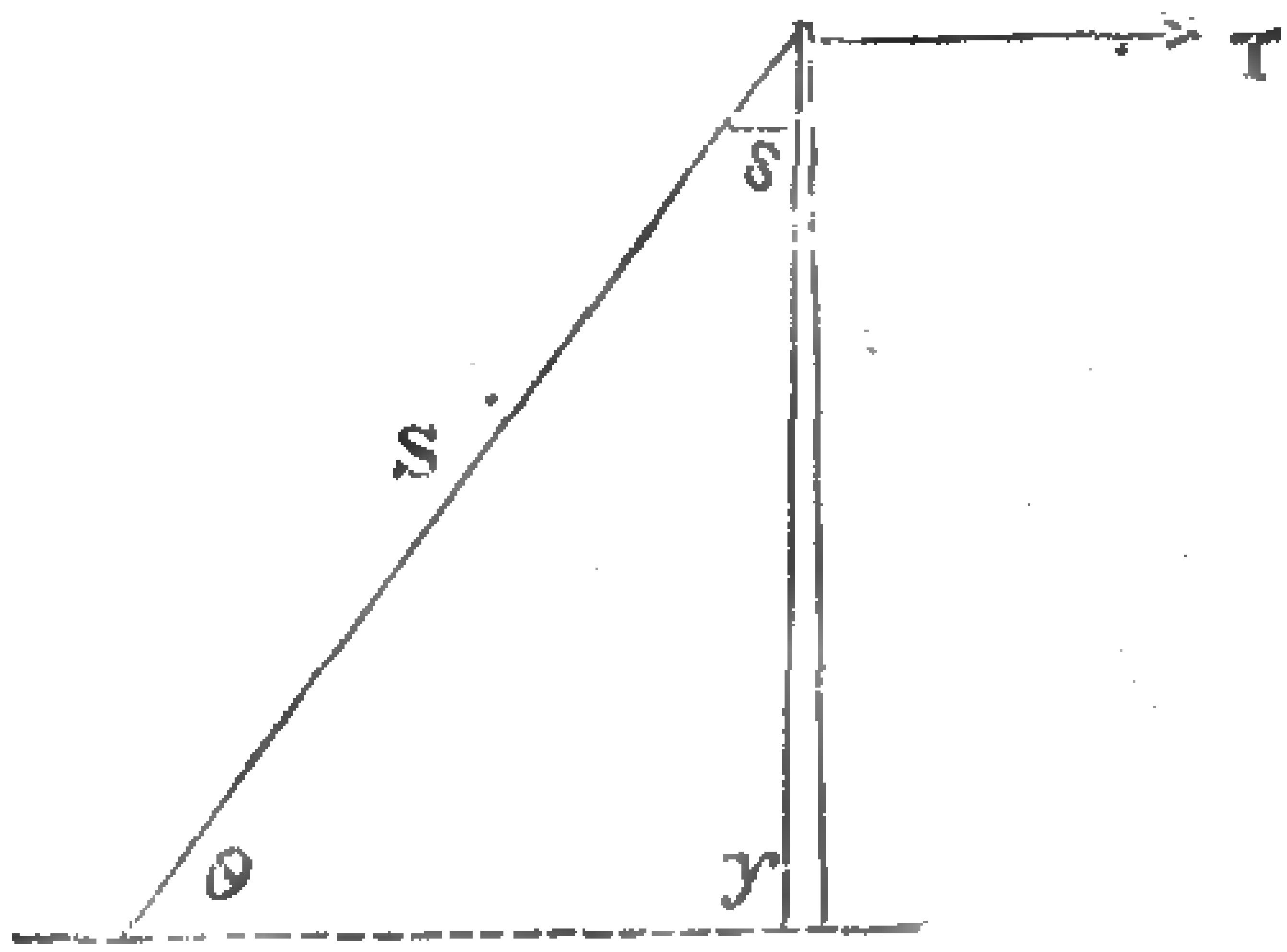
2. When the points of Support are at different levels it will be safer to take the Wind Pressure on a length of wire equal to the distance between the higher support and the lowest point of the curve that is on F E (see Second Diagram).

3. Having found this additional strain due to Wind Pressure, on the Span wire in terms of its length we must see that it is less than $\frac{3}{4}\frac{T}{W}$ or the *margin of safety*; that is less than a weight equal to 3375 yards for Steel, and less than a weight equal to 1125 yards for Iron wire.

If the additional Strain due to Wind Pressure be found to be more than $\frac{3}{4}\cdot\frac{T}{W}$. We must then increase the margin of safety and make the working Strain at the Insulator less than $\frac{1}{4}\frac{T}{W}$.

SEVENTH INVESTIGATION.

To find the strain on a Stay.



It has been assumed above that Strains due to the wire tension are balanced at the top of the Mast. There remains then (as the Mast will support the weight of the wire which acts vertically) only the Strain due to Wind Pressure.

This will be equal to almost, (and may be assumed to be equal to) the half of that bearing on the "Balancing Span" and to half that bearing on the Main Span. Adding these together and calling them T, the Strain on the Stay S, and the Angle the Stay makes with the ground θ

$$T : S : : \sin \delta : \sin \gamma$$

but as $\gamma = 90°$

$$S = \frac{T}{\sin \delta}$$

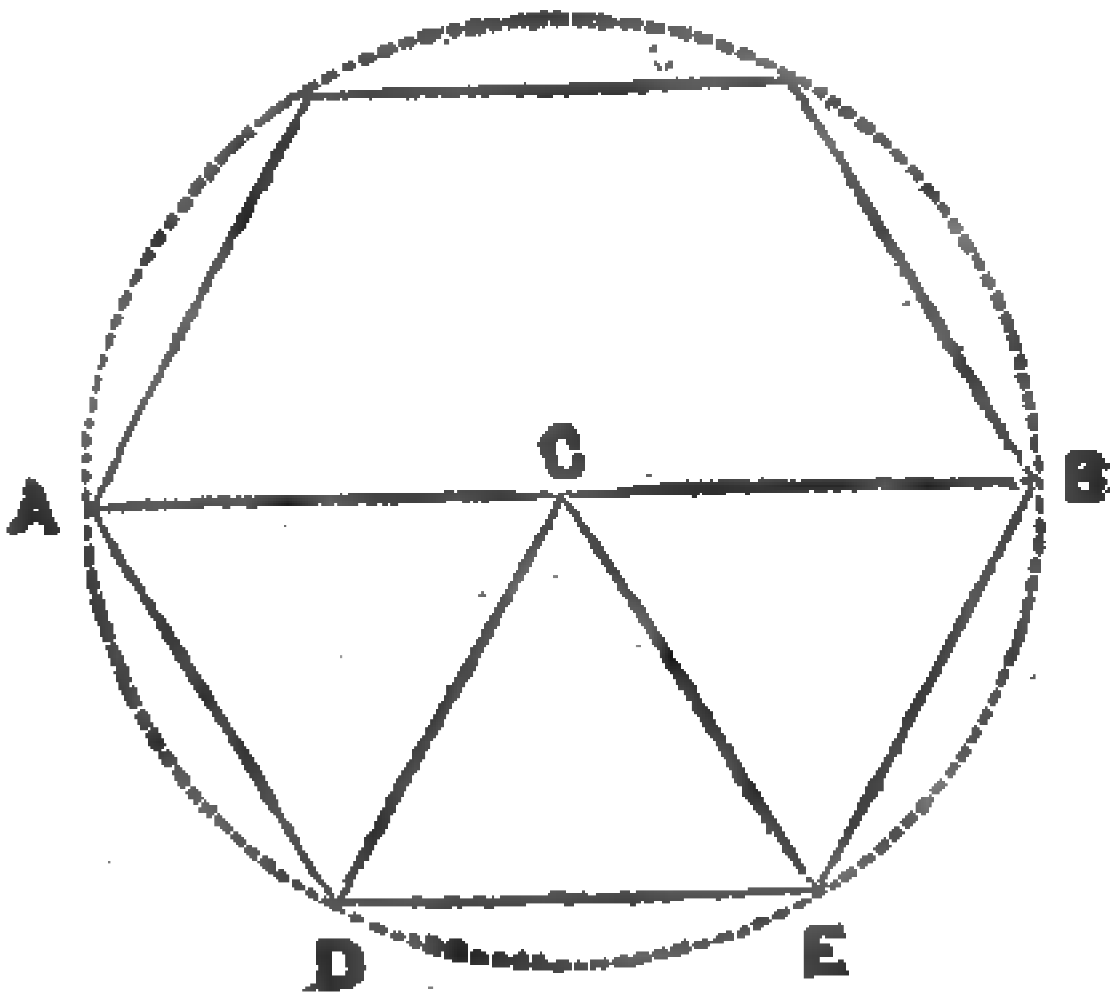
$$= \frac{T}{\cos \theta}$$

EIGHTH INVESTIGATION.

The point at which the Strain on an Angle and on a Terminal Insulator is Equal.

We have seen above that the Strain of any wire (erected in the proportion of 30 inches dip per 100 yards) on the Insulator is $\frac{5}{6}$ of its weight per mile. Therefore in completing a series of Back Spans it is evident that it is not advisable to terminate a heavy wire on any Insulator, but to relieve its Strain by a corresponding or nearly corresponding Strain on the other side of the Insulator, so that only the Resultant of two Strains may remain. But there are occasions when it is necessary to make severe angles, and it is as well to know when it is better to terminate the wire on an Insulator than to make an angle.

For this the following is a very simple proof :—



Describe a Hexagon in a Circle. Draw any radii $C\,A$, $C\,D$, $C\,E$, $C\,B$. Let C represent an Insulator, and let $B\,C\,D\,E$ represent a parallelogram of forces, of which $B\,C$ and $C\,D$ are the two equal Strains produced by 2 wires on C, and $C\,E$ the Resultant Strain.

Now the angle $A\ C\,D =$ angle $D\,C\,E =$ angle $E\,C\,B$. Hence the angle $B\,C\,D$ is 120°. But $C\,E$ the Resultant of the Strains $B\ C$ and $C\,D$ is equal to either.

That is to say

The Resultant Strain of 2 wires on an Insulator at an angle is greater or less than the Strain of a single wire terminated according as to whether the supplement of the con-

ABSTRACT.

I. Constant Strength of Iron Wire = 6000 yds. of itself.*

II. " " " Steel " = 18000 " " "*

III. With a dip in the proportion of 30″ per 100 yds. Span the Constant Working Strain of any Wire at the Insulator = 1500 " " "*

or approximately $= \frac{5}{6}$ *of its weight per mile.**

IV. The Strain varies directly in proportion to the dip.

V. With the points of support at same level.

Let Span $= a$

Dip $= d$

Length of Curve $= l$

Strain at Insulator $= T$

Strain at lowest point of curve $= T^2$

Weight of a unit of wire $= W$

Then
$$l = a + \frac{8\,d^2}{3\,a}$$

$$T_2 = \left\{\frac{a^2}{8\,d} + \frac{1}{6}\,d\right\} W$$

$$T = \left\{\frac{a^2}{8\,d} + \frac{7}{6}\,d\right\} W$$

$$\frac{T}{W} = \left\{\frac{a^2}{8}\,d + \frac{7}{6}\,d\right\}$$

$\frac{T}{W} =$ Working Strain of wire.

$$d = \frac{1}{7}\left(3\,\frac{T}{W} - \frac{\sqrt{36\,\frac{T^2}{W^2} - 21\,a^2}}{2}\right)$$

* These figures are only for the Indian Wire Specifications of 1872.

VI. With points of support at uneven levels.

Let the horizontal distance from the lowest point of the curve to the lower support $= x$

Ditto to the higher support $= y$

Difference in hight between the supports. $= a$

Parameter $= p = 2$ the Working Strain of the wire.

$$x = \frac{l^2 - a\,p}{2\,l}$$

$$y = \frac{l^2 + a\,p}{2\,l}$$

VII. To find the factors of Back Span.

Let Length of Main Span in yards $= l$

" " " Back " " " $= x$

" dip of Back Span in feet $= d$

" proportion of weight per mile of Back Span wire to that of Main Span wire $= n$

" the proportion of dip in feet per 100 yds. at which the main span wire is calculated $= a$

Then

$$x = 50 \sqrt{\frac{d}{a\,n}}$$

VIII. Let the Wind pressure per square foot $= p$ lbs.

" the weight of wire per mile $= w$ lbs.

" the length of curve $= l$ feet.

The total wind pressure on any length l feet $= \dfrac{p\,l}{2117{\cdot}16} \sqrt{w}$ lbs.

IX. To find the Strain on a Stay.

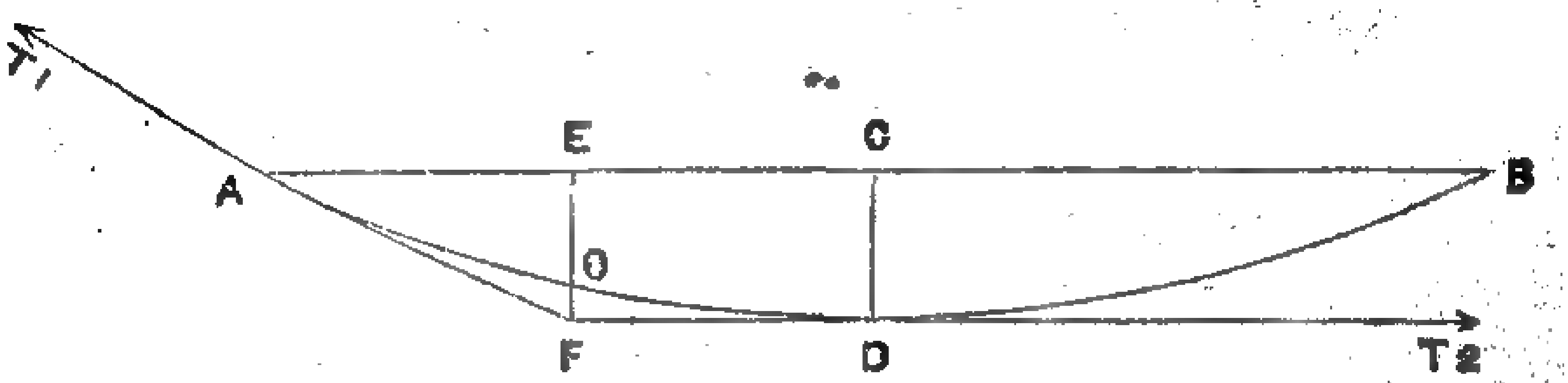
Let the Strain at the Insulator $= T$

Let the angle the Stay makes with the ground. $= \theta$

The Strain on the Stay ... $= \dfrac{T}{\text{[illegible]}}$

APPENDIX A.

Approximate proof of Leslie's equation by Major J. Eckford.



Let $A\ B$ be the chord of any Span a let $A\ 0\ D\ B$ be the curve in which the Span hangs.

Let $C\ D$ the dip $= x$

This Curve is a Catenary but the following proof is independent of the kind of curve.

Take $A\ D$, the half curve, as a figure to be considered by itself and consider it as a rigid section. This figure will be kept at rest by 3 forces *viz.* the tension T_1 acting at A, the tension T_2 acting at D, and its weight (or the weight of $A\ 0\ D$) acting vertically at 0 the centre of gravity.

Now the lines representing these 3 forces if continued will pass through the one common point F; and as their directions are each parallel to the sides of the traingle $A\ E\ F$ the magnitudes of the forces will be proportional to the sides of that traingle.

That is

$$T_2 : W :: A\ E : E\ F$$

Now in a very flat curve $A\ E$ will be very nearly but not quite equal to $\frac{1}{2}\ A\ C$ but will be very slightly less, the point O (the centre of gravity) being always nearer to A than to C.

Assuming it to be equal, $A\ E$ will equal $\frac{A\ B}{4} = \frac{\text{span } a}{4}$

and $E\ F$ will equal $C\ D = \text{dip} = d$.

Hence

$$T_2 : W :: \tfrac{1}{4}\ a : d.$$

Then

$$T_2 = W \frac{a}{4\ d}$$

But, in a very flat curve W, the weight of the curve $A\ O\ D$ will be very slightly in excess of the weight of $A\ C$ or $\frac{1}{2}\ a\ w$

assumption may balance the former assumption that $A\,O = \frac{1}{4}\,a$, we get

The tension at $D = T_2 = \frac{1}{4} \times \frac{a}{2} \times \frac{a}{d} \times w = \frac{a^2}{8\,d} \times w$

or $$\frac{T_2}{w} = \frac{a^3}{8\,d}$$

Whilst the tension at the top of the curve

$$T_1 : W :: A\,F : E\,F$$

But in the triangle AEF if we continue to assume $A\,O = O\,D = A\,E = \frac{1}{4}$ *span*

$$A\,F = \sqrt{A\,E^2 + E\,F^2}$$
$$= \sqrt{\frac{a^2}{16} + d^2}$$
$$= \sqrt{\frac{a^2 + 16\,d^2}{16}}$$
$$= \frac{1}{4}\sqrt{a^2 + 16\,d^2}$$

Hence

$$T_1 : W :: \frac{1}{4}\sqrt{a^2 + 16\,d^2} : d$$

or $$T_1 = \frac{1}{4}\,W\,\frac{\sqrt{a^2 + 16\,d^2}}{d}$$

Then substituting $\frac{1}{2}\,a\,w$ for W

we get $$T_1 = \frac{1}{4} \times \frac{1}{2}\,a\,w \times \frac{\sqrt{a^2 + 16\,d^2}}{d}$$
$$= w\,\frac{a}{8} \times \frac{\sqrt{a^2 + 16\,d^2}}{d}$$

But $$\sqrt{a^2 + 16\,d^2} = a + \frac{8\,d^2}{a}$$

Therefore $$\frac{T_1}{w} = \frac{a}{8} \times \frac{a + \frac{8\,d^2}{a}}{d}$$
$$= \frac{a^2 + 8\,d^2}{8\,d}$$
$$= \frac{a^2}{8\,d} + d$$

It is to be noticed that these proofs are within a fraction of Professor Leslie's formula which are.

The tension at the points of suspension $= \frac{a^2}{8\,d} + \frac{7}{6}\,d$

and at the bottom of the curve $= \frac{a^2}{8\,d} + \frac{1}{6}\,d$

APPENDIX B.

Indian Telegraph Wire Tests.

As the whole of the equations about Spans depend on the quality of the Wire used, this opportunity is taken of publishing the table of tests to which the special Iron Wire, now made for the Indian Telegraphs, is subjected when purchased.

2. The tests for Steel Wire differ slightly in detail but the Tensile strength of the special Steel Wire for Spans is 3 times that of Iron Wire of the same size.

3. The sizes are calculated as follows *viz.* :—

The weight of a Rod 1 foot long and 1 inch diameter $= 2{\cdot}62$ lbs.

The weight of a Rod 1 mile long and 1 inch diameter $= 13833{\cdot}6$ lbs.

The weight of a Rod 1 mile long and any diameter $= D^2 \times 13833{\cdot}6$ lbs.

Hence $$D = \sqrt{\frac{w}{13833{\cdot}6}}$$

4. The number of Twists in a given length are calculated as follows :—

Let $W =$ the weight of the Standard size.

$N =$ the number of twists in the standard size.

$w =$ the weight of any other required size.

$n =$ the number of twists there should be in any required size.

Then

$$n = N\sqrt{\frac{W}{w}}$$

5. The Tests for tensile strength are made by the direct appliance of weight vertically. No hydraulic machine or lever is used. The wire has at first to lift a weight equal at least to $\frac{9}{10}$th of the minimum tensile strength entered in the table for the size under trial, and the remaining tenth has to be added gradually, by convenient ordinary weights of not less than $\frac{1}{5}$th of the remainder, or $\frac{1}{50}$th of the minimum tensile strength. No less weight than $\frac{1}{100}$th of the minimum is reckoned.

6. Tests for ductility are made as follows :—The piece of wire is gripped by two vices, and twisted. The twists are reckoned by means of an ink spiral formed on the wire during torsion. The full number of twists must be distinctly visible between the vices, no fractions being reckoned. In sizes above 150 lbs. per mile the vices will be six inches apart, but in those of 150 lbs. or less they *may* be three inches apart.

7. IRON TELEGRAPH LINE WIRE.

Table of Weights and Tests for Standard sizes made on a Specification designed in 1872.

DIAMETER.			WEIGHT PER MILE.			WEIGHT OF 10 FEET.			TESTS FOR STRENGTH AND DUCTILITY.									
Required.	Allowed. Minimum.	Allowed. Maximum.	Required.	Allowed. Minimum.	Allowed. Maximum.	Required.	Allowed. Minimum.	Allowed. Maximum.	To bear.	Twists.	To bear.	Twists.	To bear.	Twists.	To bear.	Twists.	To bear.	Twists.
Inches.	Inches.	Inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.		lbs.		lbs.		lbs.		lbs.	
·2547	·251	·258	900	886·5	913·5	1·704	1·682	1·726	2,775	14	2,850	13	2,925	12	3,000	11	3,076	10
·3325	·229	·236	750	738·7	761·3	1·420	1·399	1·441	2,312	15	2,375	14	2,437	13	2,500	12	2,562	11
·2205	·217	·223	675	665·	685·	1·278	1·260	1·296	2,081	16	2,137	15	2,193	14	2,250	13	2,306	12
·2079	·20[illegible]	·21[illegible]	600	591·	609·	1·136	1·119	1·153	1,850	17	1,900	16	1,950	15	2,000	14	2,050	13
·1801	·177	·183	450	459·	441·	·852	·835	·869	1,388	19	1,425	18	1,460	17	1,500	16	1,538	15
·1470	·144	·150	300	294·	306·	·568	·556	·680	925	24	950	22	975	21	1,000	19	1,025	18
·1039	·102	·106	150	147·	153·	·284	·278	·290	462	17	475	16	487	15	500	14	512	13
·0735	·072	·075	75	73·5	76·5	·142	·139	·145	208	24	214	23	219	21	225	19	231	18
·0657	·065	·067	60	58·8	61·2	·113	·112	·114	167	26	171	25	176	23	180	22	185	20